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# The method of feed-in energy on disc brake squeal

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#### Abstract

Brake squeal is studied in this paper by feed-in energy analysis. Based on the brake closed-loop coupling model, a calculation method of feed-in energy for squeal mode is derived. Result of the feed-in energy indicates squeal tendency of the brake system, while formula for calculating it discloses the relation among brake squeal phenomenon and structural parameters, such as frictional coefficient, geometric shape of brake pads, elastic modulus of frictional material, substructure modal shape, etc. The method also helps to analyze the effectiveness of various structural modification schemes attempted to eliminate the squeal noise. Finally, this method is illustrated by application to a typical squealing disc brake. © 2002 Elsevier Science Ltd. All rights reserved.

## 1. Introduction

Since noise occurring in the process of braking not only destroys city environment but also lessens vehicle comfort, it is very necessary as well as profitable to eliminate it. It should be noticed referring to the associated literature that successful reduction of brake noise is achieved through structure modification of certain brake components. See for example Felske's [1] or Zhu's [2] solution to drum brake squeal, and Baba's [3] or Guan's [4] solution to disk brake squeal. Note that Felske and Baba concentrate on experimental investigation. However, Zhu and Guan carry out detailed model analysis to the brake system in addition to experimental investigation. The theory on which the model analysis in Refs. [2,4] is based is given in Refs. [5,6] and it can be generalized as follows:

(1) Brake vibration that induces noise is self-excited in nature and formed by the closed-loop coupling of the brake system.

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- (2) Generally, the squeal frequency is above several thousands, so elastic vibration modes of each brake component must be taken into consideration.
- (3) Structure design parameters of the brake components determine the brake system stability, which reflects whether the brake has a tendency to squeal definitely. Hence, it is crucial to make the structure parameters match when designing new products or when implementing structure modification to the existing brakes which squeal.

The authors of Refs. [4,7], according to the theory mentioned above, establish a closed-loop coupling model for disc brake with finite element method and modal synthetic method, and put forward a method for brake squeal analysis and the necessary structure modification. In this method, the squeal mode, namely the unstable mode obtained from the eigenanalysis of the synthetic brake model, is composed of all substructure modes. The magnitude coefficient of the substructure modal co-ordinate is defined to indicate the influence of these substructure modes on the squeal mode, and the substructure mode with the biggest magnitude coefficient is regarded as the dominant substructure mode. Then, the modal frequency of the dominant substructure mode will be set to be the target of structure modification attempted to eliminate the squeal noise. Although effective in practice, the method is not complete yet. First, the coefficient of the substructure modal co-ordinate is complex and hence considering only its magnitude is not comprehensive. Secondly, the method merely has substructure modal frequencies as modification targets, but as is known that modal shape is important as well. The above two drawbacks have to be overcome to make the analytical method more preferable.

Kinetic energy of the vehicle, which is transformed into heat through frictional interaction between brake disc and pads during the process of braking, is the unique resource of energy consumed by the brake while it is squealing. This paper just attempts to analyze the squeal problem from the view of energy. First, feed-in energy corresponding to the squeal mode has been defined and calculated based on the model in Refs. [4,7], and then the relation between calculated feed-in energy and brake squeal tendency is explored. Next, using formulae of feed-in energy, effect of design parameters and elastic vibration characteristics of substructure on squeal tendency is analyzed, and particular observation on structural modification is provided. Finally, effectiveness of the feed-in energy method is verified by an application to analysis of the same sample brake considered in Refs. [4,7].

#### 2. Calculation method of feed-in energy

The synthetic equation of the brake closed-loop coupling model is

$$M\ddot{U} + (K + K_F)U = 0, (1)$$

where U denotes the displacement vector to all nodes of the brake FE model, M and K are the discrete mass and stiffness matrix obtained by FEM and  $K_F$  is the unsymmetric connecting stiffness matrix.

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Convert the displacement vector U into the modal co-ordinate vector q of substructures of the brake system [8]:

$$U = \Phi q, \tag{2}$$

where  $\Phi$  is the matrix composed of respective mass normalized modal shape matrix of each brake substructure. Then substitute Eq. (2) into Eq. (1) to get

$$\ddot{q} + Gq = 0, \tag{3}$$

where  $G = \Phi^{T}(K + K_{F})\Phi$  is unsymmetrical but non-singular. Thus Eq. (3) has unique eigenvalues.

Then perform eigenanalysis of Eq. (3) and extract the synthetic eigenvalues and thereof corresponding right eigenvectors. Eigenvalue with positive real part forms the unstable mode, also called squeal mode, since the imaginary part is consistent with brake squeal frequency. Assuming the *i*th mode is unstable mode with eigenvalue  $\lambda_i$  and eigenvector  $\psi_i$ , the corresponding *i*th modal shape coefficient vector  $\bar{U}_i$  is defined by

$$\bar{U}_i = \Phi \psi_i. \tag{4}$$

Theoretically, the feed-in energy during one cycle of vibration should be derived from the response in physical co-ordinates. However, the brake system is non-linear in nature and will converge to periodical limit circle movement when squeal occurs, while the vibration of the synthetic model will diverge at the squeal mode because of linearization of the brake system. Therefore, response of the model cannot be used for calculation of the feed-in energy during one cycle. As a solution, we propose that the feed-in energy be derived using the magnitude and phase of the vector  $\bar{U}_i$ . The result calculated according to such proposition can be effectively used for comparative analysis of squeal tendency.

Hereinafter, the calculation process for feed-in energy will be revealed.

As shown in Fig. 1, point *a* on brake outer pad *A* and point *b* on brake disc *B* are a pair of interactive nodes in FE model, connected with a spring with stiffness *K*. We will show first how to calculate the feed-in energy during one cycle between a pair of interactive nodes. The *i*th modal shape coefficients of nodes *a* and *b* are decomposed into components  $x_a$ ,  $y_a$ ,  $z_a$ , and  $x_b$ ,  $y_b$ ,  $z_b$ , respectively, along co-ordinate axes *x*, *y* and *z*. Note that these components are complex. Then we



Fig. 1. Schematic drawing for calculation of feed-in energy between outer pad A and brake disc B.

express the relative displacement between nodes a and b as

$$x_{a} - x_{b} = A_{abx} \cos(\omega_{i}t + \theta_{abx}),$$
  

$$y_{a} - y_{b} = A_{aby} \cos(\omega_{i}t + \theta_{aby}),$$
  

$$z_{a} - z_{b} = A_{abz} \cos(\omega_{i}t + \theta_{abz}),$$
(5)

where  $A_{abx}$ ,  $A_{aby}$  and  $A_{abz}$  are magnitudes of the relative displacements along x-, y- and z-axis, and  $\theta_{abx}$ ,  $\theta_{aby}$  and  $\theta_{abz}$  are corresponding phases.

Angle  $\theta$  in Fig. 1 shows the geometrical position of nodes *a* and *b*, so we call it a geometrical position angle. Assuming the frictional coefficient  $\mu$  is constant, then the friction forces acting on nodes *a* and *b* along *x*- and *z*-axis are, respectively,

$$F_{xa} = -F_{xb} = \mu K(y_a - y_b) \cos \theta,$$
  

$$F_{za} = -F_{zb} = \mu K(y_a - y_b) \sin \theta.$$
(6)

The nodal feed-in energy  $E_{abx}$  of the node pair ab, induced by x direction frictional force during one cycle of vibration is given by

$$E_{abx} = \int_0^T F_{xa}(\dot{x}_a - \dot{x}_b) \,\mathrm{d}t$$

By substituting Eqs. (5) and (6) into the above integral, we have

$$E_{abx} = \mu K \pi A_{abx} A_{aby} \cos \theta \sin (\theta_{aby} - \theta_{abx}).$$
<sup>(7)</sup>

In the same way, we can derive the nodal feed-in energy of ab induced by z direction frictional force, as well as two components of the nodal feed-in energy between node b on disc B and node c on inner pad C

$$E_{abz} = \mu K \pi A_{abz} A_{aby} \sin \theta \sin(\theta_{aby} - \theta_{abz}),$$
  

$$E_{bcx} = \mu K \pi A_{bcx} A_{bcy} \cos \theta \sin(\theta_{abx} - \theta_{aby}),$$
  

$$E_{bcz} = \mu K \pi A_{bcz} A_{bcy} \sin \theta \sin(\theta_{bcz} - \theta_{bcy}).$$
(7')

Then by summation of the nodal feed-in energy of all node pairs between pads (A and C) and disc (B), we get  $E_{ABx}$ ,  $E_{ABz}$ ,  $E_{BCz}$ ,  $E_{BCz}$ , respectively, as well as the total feed-in energy of the brake

$$E = E_{ABx} + E_{ABz} + E_{BCx} + E_{BCz}.$$
(8)

Compared with Refs. [4,7], note that the phase information of the coefficient of substructure modal co-ordinate is neglected in Refs. [4,7]; however, the present feed-in energy method takes phase information of relative displacement between node pairs into account.

#### 3. The relationship between feed-in energy and brake squeal tendency

In order to verify whether the calculated feed-in energy can indicate the squeal tendency of the brake, the analytical method is applied to the sample disc brake investigated in Refs. [4,7]. The modal shape vector of certain mode of the bracket is multiplied by factor S so as to change the modal shape matrix, and thus bring about the system less instable. Then eigenanalysis and

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| Factor to the modal shape <i>s</i> | Squeal mode                            | λ   | Total feed-in<br>energy E | Ratio of real part of squeal mode to total feed-in energy |
|------------------------------------|--|---|---------------------------|---|
|                                    | Real part $\operatorname{Re}(\lambda)$ | Imaginary part $\operatorname{Im}(\lambda)$ |                           | $\alpha = \operatorname{Re}(\lambda)/E$                   |
| 0.91                               | 24.2                                   | 2229.7                                      | 8.52                      | 2.84 (24.2/8.52)  |
| 0.95                               | 23.1                                   | 2226.1                                      | 8.14                      | 2.84  |
| 1.0                                | 21.4                                   | 2222.5                                      | 7.50                      | 2.85  |
| 1.05                               | 18.8                                   | 2218.9                                      | 6.57                      | 2.85  |
| 1.11                               | 14.9                                   | 2215.4                                      | 5.20                      | 2.86  |
| 1.18                               | 8.4                                    | 2212.0                                      | 2.94                      | 2.86  |
| 1.25                               | Nor                                    | ne  | 0                         | _   |

 Table 1

 Relation between feed-in energy and squeal tendency

calculation of feed-in energy for the squeal mode are performed. The calculation results are listed in Table 1.

It can be seen that the imaginary part of the unstable mode representative of squeal frequency changes little, while real part representative of squeal tendency changes apparently, with change of factor S. Moreover, the real part of the unstable mode is proportional to the calculated feed-in energy (see column  $\alpha$  in Table 1). Therefore, just like real part of the unstable mode, calculated total feed-in energy E can also indicate the squeal tendency of the brake system. The more the total feed-in energy is, the more the brake tends to squeal. When the system is stable, E is zero. Although both the real part of eigenvalue and the calculated total feed-in energy can be used as the estimation indicator of squeal tendency, the later is preferable to the former since it has information of vibration characteristics involved in, and thus is more suitable in the analysis of the influence of structure parameters on squeal tendency.

#### 4. Analyzing influence of brake design parameters on squeal tendency by feed-in energy method

- 1. It can be seen from Eqs. (7), (7') and (8) that the larger the frictional coefficient, the larger the energy fed in for squealing. Thus the more frequently the squeal occurs, while keeping the system structure unchanged.
- 2. When pads with higher elastic modulus are used, namely the stiffness coefficient K in Eqs. (7) and (7') is increased, frictional force increases. This makes feed-in energy larger and squeal tendency is intensified. This conclusion agrees with the experimental result that soft frictional material benefits the brake system stability. The above two conclusions agree with those given in the paper [9].
- 3. The influence of the geometrical shape of the pad on squeal tendency can also be disclosed with the feed-in energy method. For pad long in x direction but narrow in z direction with unchanged frictional contact area, since the range of geometrical position angle  $\theta$  enlarges, frictional force along x direction is weakened such that the total feed-in energy E decreases (see Eq. (6)) and this makes the system incline stable.

# 5. Analyzing the influence of substructure elastic vibration characteristics on brake squeal tendency by feed-in energy method

It is the elastic vibration characteristics of substructure that affects the occurrence of brake squeal remarkably in practice. So it is necessary to discuss the aspect.

The squeal mode of the researched sample brake is 21.4 + 2222.5i. The feed-in energies  $E_{ABx}$ ,  $E_{ABz}$ ,  $E_{BCx}$  and  $E_{BCz}$ , and total energy E, as defined above, are calculated corresponding to the squeal mode. Results are listed in Table 2. Obviously, the following facts can be observed:

- 1. Energy fed in the brake system through the frictional interaction between pads and disc comes primarily from the x direction vibration, since  $E_{ABx}$  and  $E_{BCx}$  are much bigger than  $E_{ABz}$  and  $E_{BCz}$ .
- 2. The dominant effect of vibration of inner pad on feed-in energy is obvious, since energy fed in from frictional interaction between inner pad and disc is 2–3 times as much as that from outer pad and disc.

Now returning to formulas (7) and (7'), let us probe into the effects of geometrical position angle, as well as magnitude and phase of the relative displacement, on the calculated feed-in energy to check the above two facts.

Calculation results show that for a majority of corresponding node pairs that nodal feed-in energy induced by x direction component of frictional force is much larger than that induced by z direction component of frictional force, because: (1) relative displacement magnitude  $A_x$  is much bigger than  $A_z$  and (2) x direction components of the distributed frictional forces are much bigger than z direction ones, and moreover, x direction components point consistently into the same direction of x-axis, while z direction ones with one-to-one point oppositely so as to cause counteraction in summation. Consequently, feed-in energies  $E_{ABx}$  and  $E_{BCx}$  preponderate  $E_{ABz}$ and  $E_{BCz}$ .

As to the fact that energy fed in from inner pad and disc is much larger than that from outer pad and disc, it is ascribed mainly to the higher magnitude of relative displacement between inner pad and disc. Note that the ordering is disc, outer pad and inner pad if they are arranged in ascending order of vibration magnitude. Therefore, it can be inferred from the above facts that reduction of x direction vibration magnitude of inner pad is the most effective way to reduce feed-in energy and thus then lower the squeal level.

When vibrating at the squeal mode (a bit more above 2 kHz), brake pad will move as a rigid body in x-direction. Since brake pads are supported on the bracket, vibration characteristics of the bracket have a straightforward influence on the rigid movement of the pads and should be paid attention to. To make clear the relationship of vibration characteristics of pad and bracket, we have x-direction modal shape coefficient corresponding to squeal mode of the pairs of contact

Table 2 Feed-in energy to squeal mode

| Squeal mode  | $E_{ABx}$ | $E_{Abz}$ | $E_{BCx}$ | $E_{BCz}$ | E    |
|--------------|-----------|-----------|-----------|-----------|------|
| 21.4+2222.5i | 1.89      | 0.03      | 5.21      | 0.38      | 7.50 |

nodes on pads and bracket extracted and listed in Table 3, where nodes 547 and 553 are of inner pad, and nodes 509 and 515 are of outer pad. They correspond to the bracket nodes 1214, 1218, and 1246, 1250. These four bracket nodes position on the right arm of the bracket, shown in Fig. 2, which receives the x direction force applied by the pads. It shows clearly in Table 3 that vibration magnitudes of key nodes on inner pad are 2-3 times as much as those of key nodes on outer pad. This fact just agrees with the relationship between feed-in energy by inner pad and that by outer pad. Furthermore, the corresponding nodes of pad and bracket have almost the same modal shape, whose vibration magnitudes and phases are given in Table 3. So it can be inferred further that vibration characteristics of the right arm of the bracket. Thus, the solution of the squeal problem pivots on the understanding of the vibration characteristics of the right arm of the bracket as well as the critical factors influencing it.

By expression (4), note that the modal shape coefficient vector to squeal mode is a linear summation of all modal shape vectors of substructures.  $\bar{U}_{Ei}$ , representing the modal shape coefficient vector corresponding to the bracket, can be extracted from  $\bar{U}_i$  and expressed as a linear summation of modal shape vectors of the bracket, i.e.,

$$\bar{U}_{Ei} = \Phi_E \psi_{Ei} = \sum_j \varphi_{Ej} \cdot \psi_{Ej,i},\tag{9}$$

Table 3

x Direction modal shape coefficient to squeal mode of corresponding nodes on pads and bracket

| Nodes of pads    | Inner pad |       | Outer pad |        |
|------------------|-----------|-------|-----------|--------|
|                  | 547       | 553   | 509       | 515    |
| Magnitude        | 0.379     | 0.355 | 0.913     | 0.971  |
| Phase (deg)      | 21.3      | 22.9  | -153.3    | -154.1 |
| Nodes of bracket | 1214      | 1218  | 1246      | 1250   |
| Magnitude        | 0.413     | 0.275 | 0.896     | 1.001  |
| Phase (deg)      | 21.8      | 19.6  | -153.5    | -153.9 |



Fig. 2. Schematic drawing of key nodes on bracket.

wherein  $\Phi_E$  is the modal shape matrix of the bracket, while  $\varphi_{E,j}$  denotes the *j*th modal shape vector of the bracket.

For bracket node l, let  $\varphi_{El,j}$  be the *l*th element of the *j*th modal shape vector of the bracket. Then the modal shape coefficient of node l,  $\overline{U}_{El, i}$ , can be obtained from Eq. (9):

$$\bar{U}_{El,\,i} = \sum_{j} \varphi_{El,\,j} \cdot \psi_{Ej,\,i},\tag{10}$$

where product  $\varphi_{El,j} \cdot \psi_{Ej,i}$  is defined as *j*th bracket mode component of the modal shape coefficient  $\overline{U}_{El,i}$ . Due to their comparatively large vibration magnitude (see Table 3), bracket nodes 1246 and 1250 are considered the key nodes influencing feed-in energy and squeal tendency. Therefore, bracket mode components (BMC) of the modal shape coefficients of these two key nodes deserve calculation and further analysis. Here we have eight major BMC listed in Table 4. Since every element (i.e.,  $\psi_{Ej,i}$ ) of the eigenvector to squeal mode is complex, the calculated BMC of the modal shape coefficient are complex too. Magnitudes and phases of the eight major BMCs are given in Table 4 below.

It can be seen from the given phase information in Table 3 that bracket nodes interacting with inner pad vibrate almost oppositely to those interacting with outer pad along x direction. Moreover, based on the phase value, it can be inferred that magnitude of the bracket mode component alone can indicate influence of the bracket mode on the squeal tendency. In Table 4, the eight major BMCs are selected based on their relatively bigger magnitude. Therefore, we claim that the bracket mode with the larger magnitude of BMC exerts the strongest influence on the vibration characteristics of the key nodes. Obviously, the 11th mode of the bracket is the dominant mode affecting vibration characteristics of key nodes 1250 and 1246 according to the calculated values in Table 4, and so is taken as the structure modification target to eliminate the bracket squeal.

In Refs. [4,7], influence of substructure modes on squeal mode is estimated by their magnitude coefficients of the substructure modal co-ordinates to the squeal mode. Considering Eq. (3), let  $\Psi$  be the synthetic modal shape matrix and let r be the vector of generalized modal co-ordinates. Then we have

$$q = \Psi \cdot r, \tag{11}$$

$$r = \beta \cdot q, \tag{12}$$

| Table 4 |     |       |       |             |    |     |     |       |
|---------|-----|-------|-------|-------------|----|-----|-----|-------|
| BMC of  | the | modal | shape | coefficient | of | the | key | nodes |

| Mode order of Bracket <i>j</i> |                             | 5     | 7     | 9      | 11     | 13    | 18     | 23     | 26     |
|--------------------------------|-----------------------------|-------|-------|--------|--------|-------|--------|--------|--------|
| Node, 1250                     | Magnitude of each component | 0.145 | 0.260 | 0.239  | 0.537  | 0.070 | 0.156  | 0.112  | 0.143  |
|                                | Phase of each component     | 29.8  | 33.6  | -157.1 | -154.6 | 21.5  | -151.3 | -152.1 | -148.8 |
| Node, 1246                     | Magnitude of each component | 0.145 | 0.263 | 0.182  | 0.562  | 0.084 | 0.142  | 0.105  | 0.131  |
|                                | Phase of each component     | 29.8  | 33.6  | -157.1 | -154.6 | 21.5  | -151.3 | -152.1 | -148.8 |

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where  $\beta$  is the inversion of the synthetic modal shape matrix  $\Psi$ . Assuming  $r_i$  to be the squeal mode of the brake, it can be decomposed as

$$r_i = \sum_I \sum_j \beta_{li,j}.q_{Ij}, \quad I = A, B, C, D, E,$$
 (13)

where *I* is the identifier of brake substructure with *A* corresponding to outer brake pad, *B* to brake disc, *C* to inner brake pad, *D* to brake clipper and *E* to bracket; *j* is the order of the substructure mode; and  $\beta_{li,j}$  denotes the influence coefficient of *j*th mode of substructure *I* corresponding to the squeal mode  $r_i$ . It is a complex number. In Refs. [4,7], only magnitude of the influence coefficient, named magnitude coefficient of substructure modal co-ordinate, is used as the indicator to estimate the influence of corresponding substructure mode  $q_{Ij}$  on the squeal mode  $r_i$ .

However, in the research of the brake squeal problem hereof, elements  $(\varphi_{Il,j})$  of modal shape vector of each substructure are involved in the estimation of the dominant bracket modes and elements  $(\psi_{Ij,i})$  of the synthetic modal shape vector are also involved as they are embodied in the product  $\varphi_{Il,j} \cdot \psi_{Ij,i}$  (refer to formula (10)). Thus, the product can reflect comprehensively and straightly the influence of *j*th substructure mode on the vibration characteristics of key nodes. Consequently, it is more rational to estimate the influence of substructure mode on vibration characteristics of key nodes by the product  $\varphi_{Il,j} \cdot \psi_{Ij,i}$  than by the magnitude coefficient in Refs. [4,7].

#### 6. Study of structure modification schemes of the sample disc brake

In Ref. [4], magnitude coefficient of the substructure modal co-ordinate to squeal mode is calculated for the sample disc brake, and part of the results are given in Table 5. Then based on the estimation principle provided therein, four of the bracket modes are considered as the dominant substructure modes affecting squeal. They are the 11th mode, the 2nd mode, the 9th mode and the 8th mode. However, when having each modal frequency of these dominant substructure modification target and (then) performing structure modification analysis, it was found that the brake system is sensitive to only the 11th modal frequency of the bracket. This can be explained here by the feed-in energy method, since as is shown in Table 5 that the 11th bracket mode component of the modal shape coefficient has the largest magnitude (0.537 and 0.562, respectively, in Table 4) for the key nodes on the bracket.

In Ref. [4], two structure modification schemes to the bracket are put forward. Here we call them schemes A and B. Although both of the two modification schemes have the 11th modal

 Table 5

 Magnitude coefficients of some substructure modal co-ordinates to squeal mode

|                                 | Substructure name |             |                         |             |             |             |             |             |             |             |             |             |             |             |             |              |              |
|---------------------------------|-------------------|-------------|-------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|--------------|
|                                 | Oute              | r pad       |                         | Brake       | disc        | Clipp       | per         |             |             | Brac        | kets        |             |             |             |             |              |              |
| Mode order MEC $ \hat{a}_{ii} $ | 1st<br>0.13       | 2nd<br>0.17 | 3 <sup>rd</sup><br>0.36 | 3rd<br>0.10 | 7th<br>0.19 | 1st<br>0.15 | 2nd<br>0.15 | 3rd<br>0.17 | 5th<br>0.12 | 2nd<br>0.38 | 5th<br>0.11 | 6th<br>0.12 | 7th<br>0.16 | 8th<br>0.28 | 9th<br>0.30 | 10th<br>0.23 | 11th<br>0.42 |

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Table 6

| Squeal mode  | $E_{ABx}$ | $E_{Abz}$ | $E_{BCx}$ | $E_{BCz}$ | Ε    |
|--------------|-----------|-----------|-----------|-----------|------|
| 21.6+2231.6i | 1.77      | 0.15      | 5.19      | 0.52      | 7.63 |

Feed-in energy to squeal mode for the system modified on scheme A

Table 7

x-Direction modal shape coefficient to squeal mode of corresponding nodes on pads and bracket for system modified on scheme B

| Nodes of pads    | Inner pad |       | Outer pad |       |
|------------------|-----------|-------|-----------|-------|
|                  | 547       | 553   | 509       | 515   |
| Magnitude        | 0.341     | 0.318 | 0.485     | 0.496 |
| Phase (deg)      | 0         | 0     | 180       | 180   |
| Nodes of bracket | 1214      | 1218  | 1246      | 1250  |
| Magnitude        | 0.382     | 0.303 | 0.443     | 0.513 |
| Phase (deg)      | 0         | 0     | 180       | 180   |

Table 8

BMC of modal shape coefficient of the key nodes for brake system modified on scheme B

| Mode order of bracket <i>j</i> |                             | 5     | 7     | 9     | 11    | 13    | 18    | 23    | 26    |
|--------------------------------|-----------------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Node, 1250                     | Magnitude of each component | 0.089 | 0.480 | 0.058 | 0.103 | 0.002 | 0.142 | 0.119 | 0.028 |
|                                | Phase of each component     | 180   | 180   | 180   | 0     | 180   | 0     | 0     | 0     |
| Node, 1246                     | Magnitude of each component | 0.088 | 0.469 | 0.128 | 0.106 | 0.003 | 0.134 | 0.109 | 0.025 |
|                                | Phase of each component     | 180   | 180   | 180   | 0     | 180   | 0     | 0     | 0     |

frequency of the bracket increased by about 5%, scheme A has no contribution to restraining of brake squeal, while scheme B works effectively. Modified bracket on scheme B is used in manufacturing and effectiveness in restraining squealing is verified in Ref. [10] by field experiment. The present work makes this clear through the feed-in energy analysis.

Comparing the feed-in energy corresponding to squeal mode of the brake system modified by scheme A (shown in Table 6) with that of the original brake system (shown in Table 2), it can be seen that modification scheme A even makes energy fed in the brake system increase a bit. Hence, it cannot benefit the system stability, let alone elimination of squeal.

Next turning to the brake system modified with scheme B, we first find out the synthetic mode of the new brake model corresponding to the squeal mode of the original system, and then calculate its modal shape coefficient of the key nodes. Results are listed in Table 7.

Compared with Table 3, it can be seen that vibration magnitudes of the key nodes diminish almost by half. If we check the BMCs of the modal shape coefficient of bracket nodes 1246 and 1250 (shown in Table 8), it can be observed that the dominant bracket mode is changed significantly. That is why the modification scheme B can eliminate the brake squeal effectively.

#### 7. Conclusions

- (1) Based on the frictional closed-loop coupling model, calculation method of feed-in energy is derived. Just like the real part of the eigenvalue from the synthetic model, the feed-in energy can indicate the level of squeal tendency.
- (2) The calculation method of feed-in energy also helps to disclose the influence of structure design parameters on the feed-in energy, i.e., squeal tendency. It is concluded that small frictional coefficient, soft frictional material, or long but narrow pad shape make the brake system become more stable.
- (3) Analysis results show that magnitude of each substructure mode component of the modal shape coefficient corresponding to the squeal mode can indicate the effect of each substructure mode on squeal mode. Then based on this estimation principle, dominant substructure mode can be detected and set as the structure modification target when attempting elimination of the brake squeal.
- (4) Effectiveness of the feed-in energy method is illustrated further by application to analysis of two structure modification scheme of the sample disc brake.

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